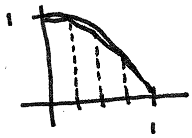
hw set 10
sect 5.9
p421

3. $\int_0^1 \cos(x^2)\,dx$

a) Trapezoid Rule $n=4$



$\Delta x = \frac{1-0}{4} = \frac{1}{4}$

underestimate

$T_4 = \frac{\Delta x}{2}\left[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+f(x_4)\right]$

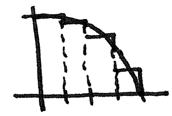
$= \frac{1}{2}\cdot\frac{1}{4}\left[\cos(0)+2\cos(\frac{1}{4})^2+2\cos(\frac{1}{2})^2 + 2\cos(\frac{3}{4})^2+\cos(1)^2\right]$

$= \cancel{[illegible]}$

$= .895758896144$

b) Midpoint Rule $n=4$

$\Delta x = 1/4$



overestimate

$M_4 = \Delta x\left[f(\bar{x}_1)+f(\bar{x}_2)+f(\bar{x}_3)+f(\bar{x}_4)\right]$

$= \frac{1}{4}\left(\cos((\frac{1}{8})^2)+\cos((\frac{3}{8})^2)+\cos((\frac{5}{8})^2) + \cos((\frac{7}{8})^2)\right)$

$= .908906790738$

c) So $\int_0^1 \cos(x^2)\,dx \approx .9045$
ie: it's b/t T_4 & M_4

15. $\int_0^3 \frac{1}{1+y^5}\,dy$, $n=6$ (6 decimal places) $\Delta x = \frac{3-0}{6} = \frac{1}{2}$

a) $T_6 = \frac{\Delta x}{2}\left[f(x_0)+2f(x_1)+\dots+2f(x_5)+f(x_6)\right]$

$= \frac{1}{2}\cdot\frac{1}{2}\left[f(0)+2f(\frac{1}{2})+2f(1)+2f(\frac{3}{2}) + 2f(2)+2f(\frac{5}{2})+f(3)\right]$

$= 1.064275$ (rounded)

b) $M_6 = \Delta x\left[f(\bar{x}_1)+f(\bar{x}_2)+\dots+f(\bar{x}_6)\right]$

$= \frac{1}{2}\left[f(\frac{1}{4})+f(\frac{3}{4})+f(\frac{5}{4})+f(\frac{7}{4}) + f(\frac{9}{4})+f(\frac{11}{4})\right]$

$= 1.067416$ (rounded)

c) $S_6 = \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2) + 4f(x_3)+2f(x_4)+4f(x_5)+f(x_6)\right]$

$= \frac{1}{6}\left[f(0)+4f(\frac{1}{2})+2f(1)+4f(\frac{3}{2}) + 2f(2)+4f(\frac{5}{2})+f(3)\right]$

$= 1.074915$

16. $\int_0^1 \sqrt{z}\,e^{-z}\,dz$, $n=10$ $\Delta x = 1/10$

a) $T_{10} = \frac{\Delta x}{2}\left[f(x_0)+2f(x_1)+\dots+2f(x_9)+f(x\right.$

$= \frac{1}{20}\left[f(0)+2f(1/10)+2f(\frac{2}{10})+\dots+2f(\frac{9}{10})+f(1\right.$

$= .372299$

b) $M_{10} = \Delta x\left[f(\bar{x}_1)+f(\bar{x}_2)+\dots+f(\bar{x}_{10})\right]$

$= \frac{1}{10}\left[f(\frac{1}{20})+f(\frac{3}{20})+\dots+f(\frac{19}{20})\right]$

$= \cancel{[illegible]}$ $.380894$

c) $S_{10} = \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+\dots+4f(x_9)+f(x\right.$

$= \frac{1}{30}\left[f(0)+4f(\frac{1}{10})+2f(\frac{2}{10})+\dots+4f(\frac{9}{10})+f(1\right.$

$= .376330$

17. $\int_0^2 e^{-x^2}dx \qquad \Delta x = \frac{2}{10} = \frac{1}{5}$

a) $T_{10} = \frac{\Delta x}{2}[f(x_0)+2f(x_1)+\dots+2f(x_9)+f(x_{10})]$

$= \frac{1}{10}[f(0)+2f(1/5)+\dots+2f(1\,4/5)+f(2)]$

$= .8818388$

$M_{10} = \Delta x[f(\bar{x}_1)+f(\bar{x}_2)+\dots+f(\bar{x}_{10})]$

$= \frac{1}{5}[f(\frac{1}{10})+f(\frac{3}{10})+\dots+f(\frac{19}{10})]$

$= .882202$

b) $|E_T| \le \frac{K(b-a)^3}{12n^2}$

$(e^{-x^2})' = -2xe^{-x^2}$

$(e^{-x^2})'' = -2e^{-x^2}+4x^2e^{-x^2}$

max $\approx .8925$

min $= -2$ (at $x=0$)

$\Rightarrow |f''(x)| \le 2 = K$

$\frac{K(b-a)^3}{12n^2} = \frac{2(2-0)^3}{12(10)^2} = \frac{16}{1200}$

So $|E_T| \le .01\bar{3}$

$|E_M| \le \frac{K(b-a)^3}{24n^2}$

So $|E_M| \le .00\bar{6}$

c) T_n & M_n w/i .00001

have $K=2$

(i) $.00001 > \frac{K(b-a)^3}{12n^2}$

$\Rightarrow n^2 > \frac{2(2)^3}{12(.00001)}$

$n > \sqrt{\frac{16}{12(.00001)}} = 365.148$

$n = 366$

for T_n to be w/i .00001

(ii) $.00001 > \frac{K(b-a)^3}{24n^2}$

$n^2 > \frac{2(2)^3}{24(.00001)}$

$n > \sqrt{\frac{16}{24(.00001)}} = 258.2$

$n = 259$

for M_n to be w/i .00001

18. $\int_0^1 \cos(x^2)\,dx \qquad \Delta x = 1/8$

a) $T_8 = \frac{\Delta x}{2}[f(x_0)+2f(x_1)+\dots+2f(x_7)+f(x_8)]$

$= \frac{1}{16}[f(0)+2f(\frac{1}{8})+\dots+2f(\frac{7}{8})+f(1)]$

$= .9023328$

$M_8 = \Delta x[f(\bar{x}_1)+\dots+f(\bar{x}_8)]$

$= \frac{1}{8}[f(\frac{1}{16})+f(\frac{3}{16})+\dots+f(\frac{15}{16})]$

$= .905619957$

b) $|E_T| \le \frac{K(b-a)^3}{12n^2}$

$f'(x) = -2x\sin(x^2)$

$f''(x) = -2\sin(x^2) - 4x^2\cos(x^2)$

max $f'' = 0$

min $f'' = -2\sin(1) - 4\cos(1)$

$\approx \boxed{-4.034296 = K}$

① $\frac{K(b-a)^3}{12n^2} = \frac{K(1)^3}{12(8)^2} = \frac{K}{768}$

So $|E_T| \le .0052529891$

② $\frac{K(b-a)^3}{24n^2} = \frac{K}{24(8)^2}$

So $|E_M| \le .0026264945$

c) T_n & M_n w/i .00001

① $.00001 > \frac{K(b-a)^3}{12n^2}$

$n^2 > \frac{K(1)^3}{12(.00001)}$

$n > \sqrt{\frac{K}{12(.00001)}} = 183.355$

$n = 184$ for T_n w/i .00001

② $.00001 > \frac{K(b-a)^3}{24n^2}$

$n^2 > \frac{K(1)^3}{24(.00001)}$

$n > \sqrt{\frac{K}{24(.00001)}} = 129.65$

$n = 130$

for M_n to be w/i .00001

20. find n such that S_n is within $.00001$ of $\int_0^1 e^{x^2}\,dx$

$f'(x) = 2x e^{x^2}$

$f''(x) = 2e^{x^2} + 4x^2 e^{x^2}$

$f'''(x) = 4xe^{x^2} + 8xe^{x^2} + 8x^3 e^{x^2}$

$= 12x e^{x^2} + 8x^3 e^{x^2}$

$f^{(4)}(x) = 16x^4 e^{x^2} + 48x^2 e^{x^2} + 12e^{x^2}$

$\min f^{(4)}(x) = 12$ (at $x=0$)

$\max f^{(4)}(x) = 76e^1$ (at $x=1$)

So $\boxed{K = 76e}$

$$.00001 \le \frac{K(b-a)^5}{180\,n^4} = \frac{76e}{180\,n^4}$$

$$\Rightarrow n^4 \le \frac{76e}{180(.00001)}$$

$$n \le \left(\frac{76e}{180(.00001)}\right)^{1/4} = 18.4$$

So $n = 20$

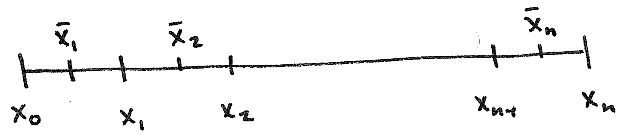
(b/c n always even for S_n)

40. Show $\frac{1}{3}T_n + \frac{2}{3}M_n = S_{2n}$

$T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$, $\Delta_n x = \frac{b-a}{n}$

$M_n = \Delta x\left[f(\bar{x}_1) + f(\bar{x}_2) + \dots + f(\bar{x}_n)\right]$, $\Delta_n x = \frac{b-a}{n}$

now for n :



then for $2n$:

So $\underset{(n)}{x_k} = \underset{(2n)}{x_{2k}}$ and $\underset{(n)}{\bar{x}_k} = \underset{(2n)}{x_{2k-1}}$

we have:

$$\frac{1}{3}T_n + \frac{2}{3}M_n = \frac{1}{3}\cdot\frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)\right] + \frac{2}{3}\Delta x\left[f(\bar{x}_1) + \dots + f(\bar{x}_n)\right]$$

$$= \frac{1}{6}\cdot\frac{b-a}{n}\left[f(x_0) + 2f(x_2) + \dots + 2f(x_{2n-2}) + f(x_{2n})\right] + \frac{2}{3}\cdot\frac{b-a}{n}\left[f(x_1) + f(x_3) + \dots + f(x_{2n-1})\right]$$

$$= \frac{1}{3}\cdot\frac{b-a}{2n}\left[f(x_0) + 2f(x_2) + \dots + 2f(x_{2n-2}) + f(x_{2n})\right] + \frac{4}{3}\cdot\frac{b-a}{2n}\left[f(x_1) + f(x_3) + \dots + f(x_{2n-1})\right]$$

$$= \frac{1}{3}\Delta x\left[f(x_0) + 2f(x_2) + \dots + 2f(x_{2n-2}) + f(x_{2n}) + 4f(x_1) + 4f(x_3) + \dots + 4f(x_{2n-1})\right]$$

$$= \frac{1}{3}\Delta x\left[f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 2f(x_{2n-2}) + 4f(x_{2n-1}) + f(x_{2n})\right]$$

$$= S_{2n}$$